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Financing entrepreneurship: Bank finance versus venture capital

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Abstract

This paper examines the entrepreneur's choice between bank finance and venture capital. With bank finance, the entrepreneur keeps full control of the firm and has efficient incentives to exert effort. With venture capital finance, there is a two-sided moral hazard problem as both the entrepreneur and venture capitalist (VC) provide unverifiable effort. The entrepreneur benefits from the VC's managerial input but must surrender partial ownership of the venture, thus diluting the entrepreneur's incentive to provide effort. Venture capital tends to be preferred to bank finance when VC productivity is high and entrepreneurial productivity is low.
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1. Executive summary

Many well-known firms, including technological leaders such as Microsoft and Intel, received venture capital financing during early-stage development. Overall, however, venture capital provides only a modest share of entrepreneurial finance and is quantitatively less important than bank finance. Furthermore, venture capital is concentrated primarily in just a few industries, with software, telecommunications, and biotechnology accounting for well over half of total venture capital investment despite accounting for less than 10% of

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GDP. Even in these industries, a majority of entrepreneurial ventures do not receive venture capital and entrepreneurs in traditional sectors such as manufacturing and retailing receive venture capital with considerably lower frequency still. Nevertheless, for entrepreneurs in all sectors, the choice of whether to seek venture capital rather than bank finance is often an important early-stage decision.

This paper focuses on the entrepreneur's choice between venture capital and bank finance. An important difference between the two is that, unlike a bank, a venture capitalist (VC) often provides substantial managerial contributions to the venture. However, this "effort" provided by the VC is not something that can be legally verified, creating a potential moral hazard problem in which the VC might provide too little effort. The entrepreneur also provides unverifiable effort and is therefore also subject to possible moral hazard. Our analysis highlights this "two-sided" moral hazard feature of venture capital finance and the importance for both the VC and the entrepreneur to have appropriate incentives to induce efficient levels of effort. The most important incentive arises from equity ownership in the venture. A higher equity share for the VC improves the VC's incentives, but it correspondingly dilutes the entrepreneur's incentives.

The alternative source of finance is bank loans, which are assumed to be available on a competitive, actuarially fair basis. Bank finance leaves the entrepreneur with full ownership of the firm, avoiding dilution of entrepreneurial effort and loss of entrepreneurial control, but it deprives the firm of the VC's managerial input. Either bank finance or venture capital finance might be preferred depending on several factors, including the specific sensitivities of effort and performance to variations in ownership structure.

One key result is that, as pure financial intermediaries, banks dominate VCs because debt contracts (offered by banks) have superior incentive properties for the entrepreneur relative to equity-based venture capital finance. Venture capital only makes sense from the entrepreneur's point of view if the VC can provide high-value managerial inputs. Interestingly, we find that it might be in the entrepreneur's interest for the VC to receive a higher ownership share than the minimum amount necessary to attract the VC. This arises when the incentive effect of additional equity on the VC is particularly important. Similarly, from the VC's point of view, making investments is attractive when the VC can make significant contributions to the venture. Venture capital is also favored by situations in which entrepreneurial effort is *not* too important.

Our paper also provides insight regarding the impact of uncertainty on the "cherry-picking" aspect of venture capital finance. Specifically, VCs have a relatively strong preference (i.e., compared to banks) for situations where ventures can do "very well" as they share fully in the upside potential, whereas banks can do no better than to have their loans repaid with specified interest.

Our analysis provides both predictive (i.e., descriptive) and normative (i.e., prescriptive) contributions. We seek to characterize a "fully rational" financial decision by an entrepreneur. To the extent that real entrepreneurs correspond to the assumption of full rationality, our analysis should provide insight into actual patterns of venture capital investment. Conversely, to the extent that actual entrepreneurs exhibit "bounded rationality" or "cognitive bias" and therefore tend to make suboptimal decisions, we would suggest that our analysis provides managerial implications or guidance that might serve to improve decision-making in entrepreneurial finance.

2. Introduction

Considerable attention has been focused on the role of venture capital in financing entrepreneurial activity, in part because many now-prominent corporations relied on venture capital finance during their early development. Well-known examples include Microsoft, Intel, and Federal Express, among others. However, venture capitalists (VCs) make only a modest contribution to the overall supply of entrepreneurial finance and, in particular, are quantitatively much less important than commercial banks. As reported in Davis (2003), fully 90% of start-ups are *not* supported by venture capital and more than 95% of small-firm financing comes from sources other than venture capital, particularly commercial banks.

Bank finance and venture capital finance are strikingly different in several respects. Most obviously, bank finance is normally in the form of loans, whereas venture capital finance consists primarily of equity[1]. Another important difference is that banks are relatively passive investors, whereas VCs normally provide managerial input to client firms. The decision of whether to pursue venture capital is typically very important for an entrepreneur. Many explicitly decide not to seek venture capital, and some entrepreneurs who receive offers of venture capital finance ultimately decline those offers.

The primary objective of this paper is to address the question of when a rational entrepreneur would seek venture capital finance in preference to relying on bank finance. In considering this question, a central feature of our analysis is the "double moral hazard" problem that arises under venture capital finance. Specifically, with venture capital finance, both the entrepreneur and the VC provide unobservable (or at least unverifiable) effort that is important to the entrepreneurial venture's performance. The key trade-off is that a higher equity share for the VC improves the VC's effort incentives but weakens effort incentives for the entrepreneur. The alternative, bank finance, leaves the entrepreneur with full ownership of firm, thus avoiding dilution of entrepreneurial effort and loss of entrepreneurial control, but it deprives the firm of the VC's managerial input. Either bank finance or venture capital finance might be preferred depending on a variety of factors, including the specific sensitivities of effort and hence performance to variations in ownership structure. From a normative point of view, we seek to offer guidelines as to when it makes sense for an entrepreneur to seek venture capital.

The literature on entrepreneurial finance has grown substantially over the past decade. Most recent studies seek to characterize financial contracts that are optimal within some fairly broad class, subject to one or more market imperfections of interest (usually of the asymmetric information type). However, given the practical importance of the choice between bank finance and venture capital, it is noteworthy that only a few papers focus explicitly on the entrepreneur's choice between these two sources of finance. Furthermore, to our knowledge, none of the papers dealing explicitly with this choice incorporate the double (or "two-sided") moral hazard problem that is the central feature of our approach.

Elitzur and Gavious (2003a) do not deal explicitly with bank finance, but they address the nearly equivalent problem of choosing between debt and equity within a venture capital

[1] See, for example, Kaplan and Strömberg (2003), who report in their Table 1 (p. 284) that approximately 80% of venture capital financing rounds in their data consisted entirely of convertible preferred equity.

contract. In contrast to our paper, Elitzur and Gavious (2003a) derive the result that optimal incentive contracts should "backload all incentive payments to the entrepreneur," implying that a straight debt contract is optimal. The key difference between Elitzur and Gavious (2003a) and our paper is their assumption of one-sided moral hazard in that only the entrepreneur provides unobservable effort. While this assumption is appropriate to their objectives (which relate primarily to dynamic or multi-period issues), the comparison with our paper is instructive. Specifically, an important contribution of our paper is that by allowing for double moral hazard we can explain the large VC equity share associated with typical venture capital contracts.

Ueda (2004) provides an insightful analysis of the contrast between bank and venture capital finance based on asymmetric information of the "hidden characteristics" type rather than on moral hazard. She assumes that an entrepreneur has private information about a project and must obtain financing. If the entrepreneur negotiates with a bank, he or she retains this private information. A financial agreement may be reached, depending in part on the potential adverse selection problem. If the entrepreneur negotiates with a VC, the VC learns the entrepreneur's private information and is potentially able to expropriate the project (in return for some compensation to the entrepreneur). One implication of this analysis is that stronger intellectual rights would make entrepreneurs more inclined to seek venture capital.

Dybvig and Wang (2002) provide a related analysis in which equity finance has a cost arising from (one-sided) effort-related entrepreneurial moral hazard, whereas debt finance gives rise to an entrepreneurial incentive to default and expropriate cash flows. This entrepreneur expropriation possibility is a near mirror-image of the Ueda (2004) framework where expropriation by the VC is a key issue.

Landier (2003) offers an analysis of bank finance and venture capital based on the assumption that there is a "stigma" that attaches to an entrepreneur if a project fails. Variations in the strength of this stigma differentially affect bargaining between an entrepreneur and either a bank or a VC, leading to a characterization of when bank or venture capital finance might be preferred. Winton and Yerramilli (2004) offer an interesting model of startup finance in which the distribution of returns (including both variance and skewness) is the crucial factor determining the dominance of either bank finance or venture capital.

Our paper is also related to the group of papers that deal with double moral hazard in entrepreneurial finance at a general level. In particular, Cassamatta (2003) addresses the interesting question as to why managerial advice and equity finance are typically "bundled" in the sense of both being provided by a VC rather than having advice provided by specialized consultants and finance provided by specialized investors. Her key point is that a contingent financial payoff (i.e., equity) provides a VC with an appropriate incentive to contribute useful advice. Inderst and Muller (2004) include two-sided moral hazard in a model that seeks to explain equilibrium dynamics in venture capital finance by focusing on two-sided search to find good "matches" in the venture capital market. Repullo and Suarez (2004) incorporate two-sided moral hazard in a model structured to explain the "stage aspect" of venture capital finance.

We note several other themes in the literature that, while not the focus of our paper, are relevant in understanding the role of venture capital finance. Amit et al. (1998) provide a general overview of VCs' role in dealing with both moral hazard and adverse selection, emphasizing their monitoring role. Elitzur and Gavious (2003b) highlight the importance of

"angels" as seed investors in startups and the moral hazard issues that arise from their interaction with entrepreneurs and venture capitalists. In Bergemann and Hege (1998), dynamic (i.e., intertemporal) risk-sharing is the key feature that determines the efficient equity share for the entrepreneur and venture capitalist, while in Hellmann (1988) and in de Bettignies (2004), the focus is on the efficient allocation of control rights between entrepreneurs and investors. Finally, we note the interesting papers by Innes (1990) and Garmaise (2001), who respectively characterize the optimal financial contract when investors and entrepreneurs are subject to limited liability constraints and when outside investors (i.e., VCs) have better information about the prospects of new ventures than entrepreneurs.

In summarizing how our paper provides "value-added" relative to the existing literature, we emphasize that our paper focuses on the double (or "two-sided") moral hazard in a model that explicitly characterizes an entrepreneur's choice between bank finance and venture capital. From our point of view, the two-sided aspect of moral hazard is fundamental to venture capital finance. Our paper therefore builds on and is consistent with the empirical and descriptive literature that emphasizes the importance of VCs' managerial contributions to their client firms. This literature includes Gorman and Sahlman (1989), Gompers and Lerner (1999), Hellmann and Puri (2000, 2002), and Brander et al. (2002), among others.

A second important contribution of our paper arises from allowing the entrepreneur to have an active role in choosing between bank finance and venture capital finance. The entrepreneur also decides on what level of equity participation to offer the VC. The VC decides whether or not to "participate" by accepting the offer. Thus both the entrepreneur and the VC have important decisions to make in our analysis. Much work on venture capital finance focuses on the VC's decision of whether and on what terms to provide finance and treats the entrepreneur as relatively passive. In our approach, the entrepreneur is an active financial decision-maker, reflecting what we view as an important part of reality.

A third important aspect of our paper, which is both a limitation and a strength, is that we abstract from a variety of important but complex issues that have been addressed by others. In particular, we abstract from various dynamic considerations that give rise to "hold-up" problems and/or ex post appropriation of assets. We also abstract from risk-aversion, adverse selection, VC monitoring, and a variety of other issues that might be of practical importance. The benefit of these abstractions is that we are able to focus clearly on what we view as the central trade-off in venture capital finance: the benefits of VCs' managerial contributions versus the cost associated with effort dilution (and loss of control) when the entrepreneur cedes significant ownership to a VC.

Section 3 describes the basic model structure, including subsections on bank finance and venture capital finance. Section 4 then compares these two types of financing so as to shed light on conditions favorable to venture capital finance. Section 5 contains an analysis of efficiency, and Section 6 is devoted to conclusions, managerial implications, and discussion. An Appendix contains proofs of propositions.

3. The model

We focus on an entrepreneur who seeks financing for a new venture. The entrepreneur has insufficient resources to fund the venture from internal sources and must therefore seek finance from an external investor. To keep the notation as simple as possible, we choose

units such that the required investment is 1. There are two potential types of outside investors: a VC and a commercial bank. To fix ideas as clearly as possible, we assume that the entrepreneur chooses one or the other but not both.[2]

The model consists of two stages. In stage 1, the entrepreneur receives financial capital from an investor (i.e., either a bank or a VC) and enters into a financial contract with the investor. Part of the entrepreneur's stage 1 decision problem is to decide whether to take venture capital finance (if it is offered) or whether to take bank finance.

In stage 2, the entrepreneurial firm may earn revenue. This revenue depends on exogenous uncertainty, on the effort of the VC (if venture capital finance is used), and on the effort of the entrepreneur.[3] In stage 2, if the investor is a bank, the entrepreneur decides on effort level e,, uncertainty is resolved, and payoffs occur. If the investor is a venture capitalist, both the VC and the entrepreneur simultaneously decide on effort levels before uncertainty is resolved and payoffs occur. Both entrepreneurial effort e and VC effort i are unverifiable, implying that contracts cannot be written contingent on e or i. The entrepreneur, the bank, and the VC are all assumed to be risk neutral.

3.1. Bank finance

Banks provide financial capital in the form of loans. The market for bank loans is assumed to be perfectly competitive. Therefore, the promised repayment or "face value" of the debt must be large enough so that the expected return to the bank is equal to the opportunity cost of the funds. Letting the risk free rate of interest equal r and recalling that the initial investment is 1, it follows that the expected repayment must be $(1+r)$. We represent $(1+r)$ by a single letter, θ. As r is positive, it follows that θ must exceed 1.

Uncertainty is of a simple form. We assume that the entrepreneurial project is successful with probability p. If the project is successful (i.e., in the "good" state), the net revenues are sufficient to pay off the face value of the debt. If the project is not successful (i.e., in the "bad" state), the net revenue of the project is 0 and the bank loan is not paid off. In order for the repayment to have expected value θ, the face value of the loan (i.e., the promised repayment in the event of success) must be θ/p.

The entrepreneur provides effort e, which will yield net revenue $R(e)$ with probability p and 0 with probability $(1-p)$. Net revenue R is revenue net of any associated production costs but before interest or debt repayment. In order to construct as clear a model as possible, we use specific functional forms. We assume that R is linear and has the form $R=\alpha e$, where α is the productivity of entrepreneurial effort in the good or successful state. The cost of effort, $c(e)$, is $e^2/2$. Net revenue R arises only in the good state (i.e., only if the project is successful) whereas cost $c(e)$ is incurred whether or not the project turns out

[2] This assumption can be relaxed. We can think of the finance considered here as being any additional finance beyond some base level of bank finance. Thus the question for the entrepreneur is whether to obtain additional bank finance or whether to use VC finance instead. With this interpretation, the analysis is essentially unchanged.

[3] Our representation of the entrepreneurial effort decision is as simple as possible. For a more detailed analysis of the entrepreneur's time allocation decision, see Lévesque and Schade (2005). Also, see Douglas and Shepherd (1999) for an analysis of the how potential entrepreneurs choose between entrepreneurship and normal employment.

to be successful. The expected payoff to the entrepreneur under bank finance, denoted E^B, can therefore be written as:

$$E^B = p(R(e) - \theta/p) - c(e)$$
$$= p(\alpha e - \theta/p) - e^2/2 \quad (1)$$

In the second stage, the entrepreneur decides on effort level e. The first-order condition for maximization is given by:

$$dE^B/de = p\alpha - e = 0 \Rightarrow e = p\alpha \quad (2)$$

The second derivative of E^B with respect to e is -1, satisfying the second-order condition for a maximum. As the entrepreneur is the full residual claimant, the efficient level of effort is chosen so that the marginal expected value of effort to the venture, $p\alpha$, is set equal to the marginal cost of effort, e.

Incorporating the optimal effort decision of the entrepreneur, the expected return to the entrepreneur under bank finance is obtained by substituting Eq. (2) into (1):

$$E^B = p^2\alpha^2/2 - \theta \quad (3)$$

We can see from Eq. (3) that α must equal or exceed $\theta\sqrt{2}/p$ if bank finance is to be feasible in the sense of providing a non-negative expected return to the entrepreneur. As $p<1$ and $\theta>1$, it follows that α also must exceed 1 for bank finance to be feasible. We refer to this condition as C1.

$$C1: \alpha > \theta\sqrt{2}/p > 1 \quad (4)$$

Proposition 1 describes the comparative static effects of the exogenous variables on entrepreneurial effort and on entrepreneurial returns under bank finance and follows from differentiation of Eqs. (2) and (3). Proofs of propositions can be found in the Appendix.

Proposition 1. *i) Under bank finance, entrepreneurial effort e is increasing in the probability of success p and in entrepreneurial productivity α.*

ii) Under bank finance, entrepreneurial payoff E^B is increasing in the probability of success p and in entrepreneurial productivity α.

3.2. Venture capital finance

Venture capital finance is more complex than bank finance. As with bank finance, we consider the second stage first (when effort decisions are made), then allow the first-stage decisions to be made with an understanding of the implications for stage 2. Both the VC and the entrepreneur anticipate the dependence of second-stage effort decisions on first-stage decisions regarding the equity share, denoted s, received by the VC in return for providing up-front financing[4]. These anticipations are incorporated in first-stage decision-making,

[4] American VCs very rarely provide debt, as found by Kaplan and Strömberg (2003). Debt is more common in Canada, as described by Cumming (2005). To keeps things (relatively) simple, we abstract from VC-provided debt.

ensuring sequential rationality (or subgame perfection) for the overall interaction between the entrepreneur and the venture capitalist.

In stage 1, the entrepreneur and the venture capitalist must decide on VC share s. If no mutually acceptable value of s can be found, the entrepreneur will either use bank finance or abandon the project. In general, either the entrepreneur or the VC might have most of the first-stage bargaining power. As a starting point, however, we assume that the entrepreneur has full bargaining power in the first stage, and is able to set s, the share of firm ownership going to the VC, subject to the participation constraint of the VC.

This assumption is most appropriate if the market for venture capital is relatively competitive. In our model, this puts venture capital finance and bank finance on the same footing in that both sources of finance must receive at least the competitive return. The difference is that the bank always receives exactly the competitive expected return, whereas the VC earns an additional expected return in some cases. Any additional return can be interpreted as a rent to the VC's scarce ability to add value to a particular venture. The opportunity cost to the VC arises from the possibility that the VC can invest the required investment at the risk-free rate. This opportunity cost is captured by θ.

As described earlier, one defining characteristic of venture capital finance is that the VC is capable of adding value through the application of effort i. The net return R to a successful entrepreneurial venture is given by:

$$R(e,i) = \alpha e + \beta i \tag{5}$$

Thus the venture capitalist can increase net revenue over and above what would be achieved by a given level of entrepreneurial effort alone.

The VC provides equity finance. Accordingly, the VC provides the initial financing requirement and receives share s of the net revenues of the firm. This equity share, s, is determined in the first stage and is therefore taken as fixed or predetermined in stage 2 when effort levels are determined.

The payoff to the venture capitalist, denoted V, is as follows:

$$V = sp(\alpha e + \beta i) - i^2/2 - \theta \tag{6}$$

where $i^2/2$ is the cost of effort. The payoff to the VC incorporates the opportunity cost of capital (θ) provided in the first stage. By stage 2, this is a sunk cost and therefore does not affect stage 2 decision-making. If the project is unsuccessful, then there is no net revenue and the VC absorbs the cost of effort i^2, and the opportunity cost of the investment. Optimal VC effort is obtained by setting the derivative of Eq. (6) with respect to i to 0 to obtain:

$$\mathrm{d}V/\mathrm{d}i = sp\beta - i = 0 \Rightarrow i = sp\beta \tag{7}$$

The payoff to the entrepreneur under venture capital finance is denoted E.

$$E = (1-s)p(\alpha e + \beta i) - e^2/2 \tag{8}$$

The entrepreneur's chosen effort level is obtained by setting $\mathrm{d}E/\mathrm{d}e = 0$ to obtain:

$$\mathrm{d}E/\mathrm{d}e = (1-s)p\alpha - e = 0 \Rightarrow e = (1-s)p\alpha \tag{9}$$

Under venture capital finance, the entrepreneur receives share $(1-s)$ of the firm's net revenues and therefore obtains only marginal benefit $(1-s)p\alpha$ from providing an extra unit

Table 1
Notation

e	Entrepreneurial effort
i	Venture capitalist (VC) effort
p	Probability of "success" (i.e., that positive net revenues are earned)
θ	Opportunity cost of capital
s	Share of the firm owned by the venture capitalist
E^B	Entrepreneur's payoff under bank finance
E	Entrepreneur's payoff under venture capital finance
V	VC's payoff
α	Marginal value of entrepreneurial effort in the successful state
β	Marginal value of VC effort in the successful state

of effort rather than the firm's full marginal benefit, $p\alpha$. Correspondingly, the entrepreneur provides less effort than would be provided under bank finance. In the introduction, we emphasize both ownership dilution and loss of entrepreneurial control as causes of reduced or less effective managerial effort. We view both these effects as operating through s. Specifically, an increase in s (i.e., a reduction in the entrepreneur's ownership) might cause a literal reduction in effort. In addition, the associated loss of control might lead to less effective effort, which also would be reflected in reduced "efficiency units" of entrepreneurial effort. For our purposes, there is little to be gained by treating control loss and effort dilution separately so we combine them in this way.

The notation of the model is summarized in Table 1.

The comparative static properties of the effort choices are described in Proposition 2.

Proposition 2. *i) Entrepreneurial effort under venture capital finance is less than under bank finance for given levels of exogenous and predetermined variables.*

ii) Entrepreneurial effort is increasing in the probability of success p and in entrepreneurial productivity α. Entrepreneurial effort falls as the ownership share s of the venture capitalist increases.

iii) VC effort is increasing in the probability of success p, in VC productivity β, and in the VC ownership share s.

The central "trade-off" for the entrepreneur in seeking venture capital finance is as follows. The benefit is that the VC can provide managerial value-added to the firm. The cost is that the entrepreneur must surrender partial ownership of the firm and therefore face diluted incentives in providing effort. Moreover, as we shall see in the "unconstrained" case below, the entrepreneur also may bear the cost of forfeiting future rents to the VC. Weighing the relative importance of these factors influences the terms of any agreement between the VC and the entrepreneur and determines whether venture capital finance or bank finance will be chosen.

3.3. Equity shares under venture capital finance (unconstrained case)

The participation constraint for the VC is that the expected payoff V must be non-negative. Starting with expression (6) for V and then substituting from Eqs. (7) and

(9) for e and i respectively yields the following form for the VC participation constraint:

$$V = sp(\alpha e + \beta i) - i^2/2 - \theta$$

$$= p^2\alpha^2 s + p^2(\beta^2 - 2\alpha^2)s^2/2 - \theta \geq 0 \tag{10}$$

In stage 1, the entrepreneur chooses the optimal ownership share s for the VC. This is the value of s that maximizes the entrepreneur's expected return E, subject to participation constraint (10). The expected first-stage payoff to the entrepreneur is obtained by taking the expected payoff given by expression (8) and substituting in the optimal second-stage levels of e and i from expressions (7) and (9), respectively. The resulting stage 1 objective function is given by:

$$E = (1-s)p(\alpha e + \beta i) - e^2/2$$

$$= p^2(1-s)(\alpha^2 + 2s\beta^2 - s\alpha^2)/2 \tag{11}$$

We focus attention first on what we refer to as the "unconstrained" solution for s, which arises if E is maximized without regard for the VC's participation constraint (10). If this unconstrained solution allows the VC to cover the opportunity cost of capital (i.e., if the participation constraint is satisfied), then this unconstrained value for s is the overall solution. If the unconstrained value of s does not satisfy participation constraint (10), then we must consider a constrained solution, which is described in the next subsection. The unconstrained solution for s is characterized by the following first and second-order conditions:

$$\text{FOC} : \mathrm{d}E/\mathrm{d}s = 0 \Rightarrow s = (\beta^2 - \alpha^2)/(2\beta^2 - \alpha^2) \tag{12}$$

$$\text{SOC} : \mathrm{d}^2E/\mathrm{d}s^2 < 0 \Rightarrow \alpha^2 - 2\beta^2 < 0 \Rightarrow \text{C2} : \beta > \alpha/\sqrt{2} \tag{13}$$

It is helpful to consider Fig. 1, which shows three possible shapes for the entrepreneur's payoff function E as a function of VC share s. These three shapes reproduce simulations of Eq. (11) for particular parameter values of α, β, and p done using Maple 9.5.

The VC's share is restricted to be between 0 and 1. Accordingly, an apparent maximum to the left of the vertical axis implies a corner solution at $s=0$. A VC share of 1 implies a return of 0 to the entrepreneur regardless of other parameter values, so all three objective functions meet the horizontal axis ($E=0$) at $s=1$. If $\beta \leq \alpha/\sqrt{2}$, second-order condition (13) is not satisfied. This is illustrated by the (downward sloping) dashed payoff function in Fig. 1. (The actual parameter values used for the dashed payoff function in Fig. 1 are $\alpha=4$, $\beta=2$, and $p=0.75$.) In this case, any increase in the VC's share reduces the return to the entrepreneur and the unconstrained optimum occurs at the corner solution where $s=0$. As the venture capitalist would get nothing if $s=0$, the venture capitalist's participation constraint would not be satisfied.

Consider next the dotted payoff function, which has an apparent local maximum at $s<0$. This occurs if $\alpha > \beta > \alpha/\sqrt{2}$, as implied by first-order condition (12) and second-order

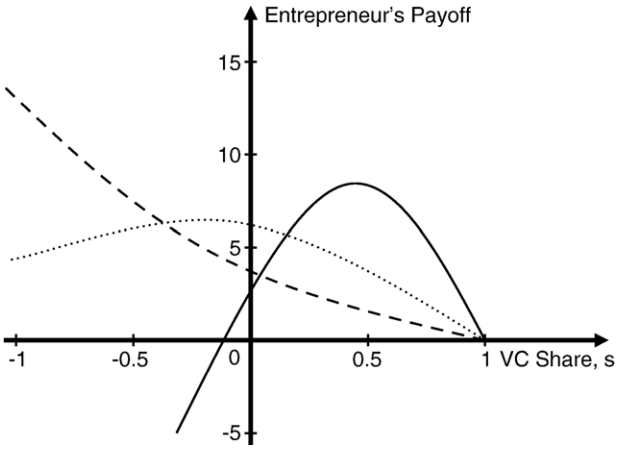

Fig. 1. Possible payoff functions for the entrepreneur.

condition (13). (The parameter values used for the dotted payoff function are $\alpha=4$, $\beta=3.4$, and $p=0.9$.) Once again, considering the requirement that s be non-negative, this implies that the unconstrained optimum for the entrepreneur would occur at $s=0$. Both the dashed and dotted objective functions in Fig. 1 illustrate situations where $\beta<\alpha$, in which case the entrepreneur's unconstrained choice for s would be $s=0$ and therefore would not be sufficient to attract venture capital finance.

The solid payoff function has a local and global maximum where $s>0$. (The parameter values used for this case are $\alpha=4$, $\beta=9$, $p=0.6$.) More generally, a solution for s between 0 and 1 arises if and only if $\beta>\alpha$, which we refer to as condition C3.

$$\text{C3}: \beta>\alpha \tag{14}$$

If C3 holds, then the entrepreneur's choice of s, if unconstrained by the VC's participation constraint, would be strictly positive. Note also that condition C3 implies that C2, the second-order condition, also must hold.

The striking feature of this interior solution is that it demonstrates that it is *not*, in general, in the entrepreneur's interest to offer the VC as low a share as possible. As illustrated by the solid objective function in Fig. 1, the entrepreneur would prefer to provide the VC with an equity share of 40% rather than, for example, one of 20%, even if this is more than needed to satisfy the VC's participation constraint. It is in the entrepreneur's interest for the VC to have a high enough share to induce efficiently high effort levels from the VC. This benefit of a higher value of s must be traded off against the dilution effect on the entrepreneur's own effort and against the lower share of net income going to the entrepreneur.

If the entrepreneur's unconstrained optimal choice for s more than satisfies the VC's participation constraint, then this constraint is not binding. This outcome is similar to the "efficiency wage" outcome sometimes discussed in labor economics, in which employers provide more than the reservation wage to employees so as to make them more productive. In this case, the entrepreneur would be willing to provide more than the minimum necessary

equity share to the VC in order to generate incentives for the VC to contribute additional managerial effort. As previously noted, in this case the VC earns a rent (i.e., an above-normal return) on its scarce ability to add value. Proposition 3 reports important properties of the unconstrained interior solution for s.

Proposition 3. *If the VC participation constraint (10) is not binding,*

i) the VC share s is decreasing in entrepreneurial productivity α.
ii) the VC share s is increasing in VC productivity β.
iii) the maximum VC share is 50%.

Proposition 3 has a natural interpretation. As the marginal productivity of the VC becomes relatively more important, motivation of the VC becomes increasingly valuable to the entrepreneur, leading to a higher preferred choice for s. However, the maximum value of s the entrepreneur would choose is 50%.

One implication of expression (10) is that it is not always in the VC's interest to have a higher value of s. Just as the entrepreneur obtains a benefit from strong effort incentives for the VC, the VC gains a benefit from strong effort incentives for the entrepreneur. However, the VC's payoff is increasing in s under certain highly relevant conditions, as expressed in Proposition 4.

Proposition 4. *i) The payoff to the VC is increasing in s at the unconstrained solution for s.*
ii) The VC's payoff is increasing for all s if condition C3 holds.

3.4. Choosing the VC's equity share with a binding participation constraint

We now consider the case in which the VC's participation constraint (given by Eq. (10)) is binding. We start by considering when the participation constrained is binding as opposed to when it is satisfied at the unconstrained solution for s.

Proposition 5. *i) The participation constraint for the venture capitalist will be satisfied at the entrepreneur's unconstrained choice for s given sufficiently high VC productivity* β.
ii) Increases in the opportunity cost of capital θ *tend to reduce the likelihood of satisfying the VC's participation constraint in the sense that* $dV/d\theta < 0$.
iii) Increases in p make it more likely that the VC's participation constraint is satisfied in the sense that $dV/dp > 0$.

Proposition 5 provides information about when an unconstrained solution for s will occur. High values of VC productivity β favor an unconstrained solution for s. In addition, a low opportunity cost of capital θ and a high probability of project success p also favor an unconstrained solution for s because, other things being equal, they increase the VC's payoff. The effect of changes in entrepreneurial productivity α is not specified in Proposition 5, as this is generally ambiguous. On the one hand, high values of α generate more surplus, which favors an unconstrained solution. On the other hand, high values of α also lead the entrepreneur to choose lower values of s, which tends to make it less likely that an unconstrained solution for s will attract the VC.

We now characterize the solution for s when the VC's participation constraint is binding. A constrained solution for s implies that condition (10) holds with equality. As Eq. (10) is a quadratic equation in s, we can apply the quadratic formula to obtain the two solutions for s as follows.

$$s = (\alpha^2 p \pm (p^2\alpha^4 - 4\theta\alpha^2 + 2\theta\beta^2)^{1/2})/(p(2\alpha^2 - \beta^2)) \tag{15}$$

The solutions for s given by Eq. (15) are feasible possibilities for venture capital finance only if they are real numbers between 0 and 1. It is, however, instructive to consider initially several unrestricted possibilities for these solutions. Fig. 2 illustrates three possible shapes for V as a function of s and allows us to see the solutions for s, which occur where V equals 0 and therefore cuts the horizontal axis. This diagram reproduces a simulation generated by Maple 9.5 for particular sets of parameter values.

One possibility is illustrated by the dotted line. In this case, there is no value of the VC's share that will satisfy the VC's participation constraint. This occurs if the discriminant in Eq. (15), given by $(p^2\alpha^4 - 4\theta\alpha^2 + 2\theta\beta^2)$, is negative, implying that there are no real solutions for s. In this case, venture capital finance would never be offered. A second possibility is shown by the dashed line. In this case, the discriminant is positive and there are two real solutions for s, one of which is negative. Therefore, only one solution is relevant. (The negative solution would imply that the VC "short sells" equity in the firm to the entrepreneur. We rule out this possibility.) As drawn, the positive solution in this case implies a VC ownership share of approximately 40%.

Finally, the solid line shows a case in which there are two positive solutions, only one of which lies between 0 and 1. Only the solution between 0 and 1 is feasible. There is also a fourth possibility, in which both solutions for s lie between 0 and 1. In this case, the entrepreneur would choose the solution yielding the highest payoff.

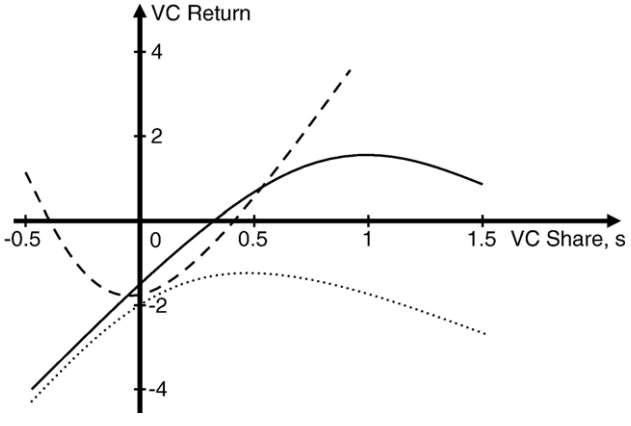

Fig. 2. Possible payoff functions for the venture capitalist.

Proposition 6 characterizes the choice between constrained and unconstrained outcomes for the VC equity share subject to condition C2 and to the following additional condition that ensures that the entrepreneur is reasonably productive.

$$\text{C4}: \alpha > 2\sqrt{\theta/p}$$

Proposition 6. *Under conditions C2 and C4, there exists a threshold level of β, denoted β*, such that:*

i) For all values of β in the range $(\alpha/\sqrt{2}, \beta^*)$*, the unique solution to the venture capital finance problem is the constrained optimum* $s = (\alpha^2 p - (p^2\alpha^4 - 4\theta\alpha^2 + 2\theta\beta^2)^{1/2})/(p(2\alpha^2 - \beta^2))$.

ii) For all values of $\beta \geq \beta^*$*, the unique solution to the venture capital finance problem is the unconstrained optimum* $s = (\beta^2 - \alpha^2)/(2\beta^2 - \alpha^2)$.

The intuition behind Proposition 6 is as follows. When the VC's productivity is low, the entrepreneur is unwilling to relinquish a large equity stake to the VC, first because the benefit of stronger VC incentives is low and second because there is a cost in the form of diluted ownership. When β is in the range $(\alpha/\sqrt{2}, \beta^*)$, the equity stake that the entrepreneur would like to give the VC (the unconstrained solution) is too low to attract the VC. Thus, the entrepreneur must give a larger stake than he or she would choose in the unconstrained case in order to induce participation from the VC.

When the VC's productivity is high, on the other hand, the incentive benefits from relinquishing a large equity stake to the VC are large. As previously noted, in this case the entrepreneur gives up more equity than is minimally necessary for VC participation in order to enhance incentives for the VC. The VC therefore receives a rent or "efficiency wage."

Proposition 6 characterizes the solution provided that venture capital finance is chosen. It does not state when or whether venture capital finance would be chosen by the entrepreneur in preference to debt finance. This question is taken up in Section 4.

4. Bank finance or venture capital?

The primary purpose of this paper is to determine when venture capital finance will arise and when bank finance will arise. Venture capital finance will be chosen by the entrepreneur if it is feasible and if it dominates bank finance. Formally, this occurs if the entrepreneur's return from venture capital finance E exceeds the entrepreneur's return from bank finance E^B. Using Eqs. (3) and (11), the difference between E and E^B can be written as:

$$\Delta E = E - E^B = p^2(1-s)(\alpha^2 + 2s\beta^2 - s\alpha^2)/2 - (p^2\alpha^2/2 - \theta) \tag{16}$$

By considering how ΔE is affected by variations in parameter values, we can assess when venture capital finance will be chosen. Proposition 7 provides the main insight regarding this point.

Proposition 7. *i) If condition C1 holds ($\alpha > \theta\sqrt{2/p}$), implying that bank finance is feasible, then there exists a threshold level of β, denoted β**, such that bank finance dominates venture capital finance for $\beta < \beta^{**}$ and venture capital finance dominates bank finance for $\beta \geq \beta^{**}$.*

ii) If there is a feasible unconstrained solution for s, then the advantage of venture capital over bank finance is increasing in probability of success p, opportunity cost of capital θ, and VC productivity β. It is decreasing in entrepreneurial productivity α.

One interesting result in Proposition 7 is that the advantage of venture capital finance increases with p. The reason is as follows. Banks earn the competitive expected return irrespective of the probability of success. (Low probability projects must offer a high enough nominal interest rate to offset the low chance of success.) On the other hand, VCs have a differential preference for projects with high success probabilities because they share the surplus above opportunity costs for cases in which the probability of success is sufficiently high that the unconstrained solution for s is feasible.

Another interesting result of Proposition 7 is that the relative advantage of equity finance also increases with β. Fig. 3 shows how changes in VC productivity affect financing options.

The parameter values used in Fig. 3 are $\alpha = 3$, $p = 0.7$, and $\theta = 1.1$. The dotted line shows the entrepreneur's payoff under bank financing. This is unaffected by the value of β as VC effort has no role under bank finance. For the parameter values considered here, debt finance is feasible in that it generates a positive expected return to the entrepreneur.

The dashed line shows the return to the entrepreneur if s is set at its optimum without regard for the VC's participation constraint. As illustrated, this value of s is not enough to generate a positive expected return to the VC if β is less than 3.9. If the entrepreneur

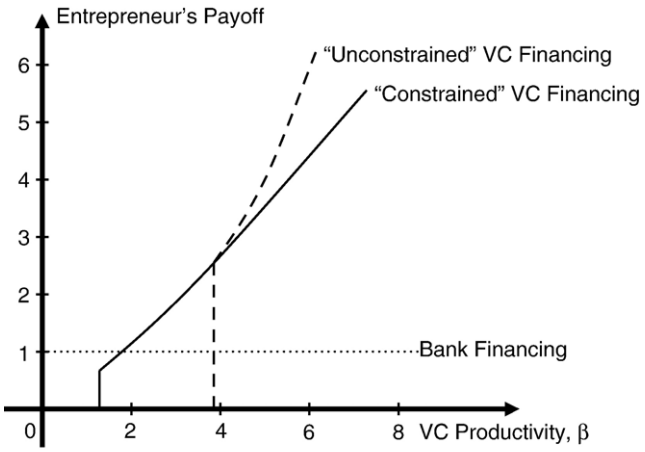

Fig. 3. Comparison of financing regimes as VC productivity varies.

Table 2
Conditions affecting mode of finance

1. $\alpha < \sqrt{2\theta/p}$	Bank finance is not feasible
2. $\alpha \geq \sqrt{2\theta/p}$	Bank finance is feasible
3. $\alpha = 0$	Venture capital finance is feasible if $\beta > \sqrt{(2\theta)/p}$
4. $\beta = 0$	Bank finance is preferred
5. $\beta < \alpha$	VC cannot earn surplus

sets s at the unconstrained level, the VC participates only for values of β exceeding 3.9. The solid line shows the return to the entrepreneur if s is chosen precisely to set the VC payoff (net of the opportunity cost of capital) to 0. For values of β less than 1.2, there are no values of s that can achieve this result.

The overall optimal financing strategy is given by the upper envelope of the three payoff functions shown in Fig. 3. In this case, bank finance is preferred for VC productivity levels (β) less than 1.5. For VC productivity between 1.5 and 3.9, venture capital finance is preferred with the VC participation constraint binding (so $V=0$). For VC productivity higher than 3.9, venture capital finance is still preferred and the VC's equity share is large enough to allow the VC to earn a net surplus.

Fig. 3 provides one particular simulation illustrating how changes one particular variable (VC productivity) change comparative returns to bank finance and VC equity finance. More generally, while the interactions between the variables are complicated, it is possible to derive some analytical conditions concerning financial choices, as shown in Table 2.

Conditions 1 and 2 in Table 2 follow directly from Eq. (4). Condition 3 arises from allowing α to equal 0 in expression (15). In this case, the entrepreneur's only contribution is the property right over the basic idea. If the VC is sufficiently productive in applying this idea, then venture capital finance is feasible. Bank finance is clearly not feasible in this case. Condition 4 shows that the VC will never be a "pure" financier in that bank finance is preferred if $\beta=0$. In this case, the entrepreneur would have to offer some positive level of s in return for the startup capital requirement and would therefore suffer some effort dilution. Actuarially fair bank finance would therefore always be preferred as no effort dilution would occur. As implied by expression (12), the VC cannot earn surplus unless β exceeds α.

One key theme is that, in order for bank finance to arise, the entrepreneur must be sufficiently productive on a stand-alone basis (i.e., α must be sufficiently high). If the entrepreneur is not sufficiently productive on a stand-alone basis for bank finance, venture capital finance is possible if the VC is sufficiently productive. A second key theme is indicated by Fig. 3 and Proposition 7 (ii). The main consideration in determining the mode of finance is the relative productivity of the VC and the entrepreneur. For any given level of entrepreneurial productivity, low values of VC productivity will lead to either no finance (if α is too low) or to bank finance (if α is sufficiently high). Higher levels of β allow for venture capital finance, and sufficiently high levels of β allow the VC to earn rents.

5. Efficiency

The agency problem inherent in our model arising from non-verifiability of effort would be expected to have an efficiency cost. We might consider two levels of increased

efficiency. Maximum efficiency could be obtained if it were possible to determine the optimal levels of e and i directly and require the entrepreneur and the VC to provide these effort levels as if it were possible to observe effort and write costless enforceable contracts on effort. We refer to this as the first-best effort allocation. A second level of efficiency, referred to as the second-best allocation, arises if effort levels are not verifiably observable but if some overall decision-maker can set s to maximize aggregate benefits to the entrepreneur and VC without regard to participation constraints.

The combined expected return to the VC and entrepreneur, denoted W, is the sum of E and V.

$$W(e,i) = E + V = p(\alpha e + \beta i) - e^2/2 - i^2/2 - \theta \tag{17}$$

The first-best levels of e and i are obtained by setting the partial derivatives of W with respect to e and i to 0 to obtain the following.

$$e = p\alpha \quad i = p\beta \tag{18}$$

Overall benefits are obtained by substituting Eq. (18) into Eq. (17).

$$W = p^2(\alpha^2 + \beta^2) - \theta \tag{19}$$

The first best levels of e and i can be compared with the levels that emerge in the market equilibrium. These levels are given by Eqs. (7) and (9) as repeated here.

$$e = (1-s)p\alpha \quad i = sp\alpha \tag{20}$$

As can be seen by comparing Eqs. (18) and (20), the equilibrium levels of e and i are strictly less than the first-best levels for any value of s between 0 and 1.

We now consider the second-best outcome. To find the value of s that maximizes W, we substitute the expressions from Eq. (20) for e and i into Eq. (17) to obtain:

$$W(s) = p^2\alpha^2(1-s^2)/2 + p^2\beta^2 s(1-s/2) - \theta. \tag{21}$$

The first-order condition for maximization of W is given by:

$$\text{FOC}: \mathrm{d}W/\mathrm{d}s = p^2(\beta^2 - \alpha^2 s - \beta^2 s) = 0 \Rightarrow s = \beta^2/(\alpha^2 + \beta^2) \tag{22}$$

The second-best value for s can be compared with the equilibrium value for s in the unconstrained equilibrium case, given by Eq. (12), and in the constrained case, given by Eq. (15). One useful comparison is provided by allowing β to grow large, in which case the unconstrained value given by Eq. (12) is the relevant equilibrium solution for s. This value of s approaches an upper limit of 1/2, in contrast to the second-best value of s given by Eq. (22), which approaches 1 as β gets very large. Fig. 4 depicts total payoffs or benefits in the first-best case (shown by the solid line), the second-best case (shown by the dashed line), and the market equilibrium (shown by the dotted line) as β changes (holding other parameters fixed at particular values).

The agency problem in our model generates two types of inefficiency. One type of inefficiency arises from double-sided moral hazard under equity (i.e., venture capital) finance. The sharing rule implied by equity financing is such that both the VC and the entrepreneur cannot *at the same time* have a claim on 100% of expected rents. Thus, one or both must exert less than first-best effort, even if s is set at the best possible level. This inefficiency is

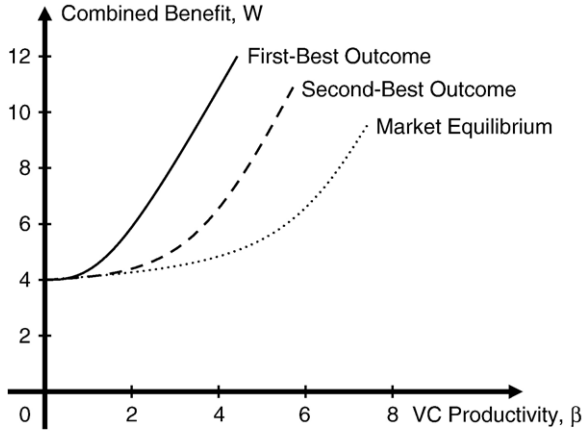

Fig. 4. Efficiency as VC productivity varies.

represented by the difference between the solid line and the dashed line and is the difference between what we call the "first best" and "second best" outcomes.

The second inefficiency is that s is not set at the best level (the level that maximizes joint returns) because of the market power of the entrepreneur combined with the entrepreneur's inability to extract all expected or *ex ante* rents using upfront transfers. If such *ex ante* transfers (which we implicitly rule out) were available, the entrepreneur would set s to maximize total return and extract all rents. In other words, the entrepreneur would choose the second-best s, and combined payoffs would be as depicted by the dashed line in Fig. 4. However, because the entrepreneur cannot extract rents *ex ante*, any equity given up to elicit more VC effort is forfeited. In view of that cost, the entrepreneur will forfeit too little equity compared to the second-best case. The entrepreneur is much like a monopsony employer who pays a wage that is inefficiently low. This inefficiency is represented by the difference between the dashed line and the dotted line and is the difference between the "second best" outcome and the equilibrium outcome in our model.

Both inefficiencies fall as VC productivity falls. This is not surprising. The less important the VC's input, the less important it is to provide the VC with strong incentives, which reduces both types of inefficiency. At the extreme where $\beta = 0$, the inefficiencies disappear altogether.

6. Conclusions, managerial implications, and discussion

This paper focuses on an entrepreneur's choice between venture capital finance and bank finance. A key insight of our analysis is that the desirability of venture capital finance depends largely on the venture capitalist's ability to provide managerial contributions to the venture. In our analysis, the VC cannot survive as a pure financial intermediary; bank finance would always be preferred to a VC who could not provide managerial value-added to the venture.

Venture capital finance has the advantage of allowing for the VC to make significant managerial contributions to the firm. However, the corresponding shift of partial ownership

and control of the firm from the entrepreneur to the VC weakens the entrepreneur's incentive to provide effort. Given particular levels of the probability of success and the opportunity cost of capital, the choice between venture capital and bank finance is determined by the trade-off between VC productivity and the entrepreneur's effort dilution. Higher levels of VC productivity favor the use of venture capital finance. Thus, we expect to see venture capital finance in cases where the VC can provide high levels of managerial value-added.

We believe that this conclusion is consistent with one very important empirical reality in venture capital finance: the high concentration by industry. For example, as reported by PriceWaterhouseCoopers Moneytree at www.pwcmoneytree.com, approximately 22% of all venture capital invested in the US in 2005 went into software (which accounts for less than 3% of GDP) and another 17% went into biotechnology (which accounts for less than 2% of GDP). Conversely, retailing and distribution (as well as the broad industrial sector, including manufacturing) have GDP shares of about 13% and 18%, respectively, but receive only trace amounts of venture capital. When queried about this pattern, VCs typically respond by observing that areas like software and biotechnology are areas where venture capitalists have considerable specialized knowledge, unlike retailing or manufacturing, where they typically have relatively little managerial value to offer. Thus, VCs do concentrate in areas where their managerial input is most important rather than being general financial intermediaries.

In order to offer clear and intuitive results, we propose a simplified and stylized model that abstracts from important features of the "real world." First and foremost, we assume that market participants are "fully rational," including having the capacity to look ahead as required by the sequentially rational (or "subgame perfect") Nash equilibrium concept. As noted in many studies including, for example, Cooper et al. (1995) and Busenitz and Barney (1997), there is considerable evidence that entrepreneurs and other market participants are prone to both "bounded rationality" (i.e., limits on their ability to process information sufficiently to make fully rational decisions) and to "cognitive biases" (i.e., systematic deviations from full rationality). Accordingly, we view our analysis as only partially descriptive and at least as partially normative or prescriptive. More specifically, we believe that we capture some important aspects of real markets and provide a useful explanation as to when venture capital finance might be expected in preference to bank finance. However, we recognize that actual decisions often fall short of full rationality and emphasize that our analysis suggests specific insights that might be helpful to entrepreneurs (and others) in making financial decisions.

Specifically, our analysis yields three primary managerial implications. First, from the perspective of the entrepreneur, we suggest that venture capital is most useful when the entrepreneurial venture lies within the venture capitalist's area of managerial expertise. Unless the venture capitalist can provide significant managerial input, bank finance is probably preferable. Furthermore, it might be in the entrepreneur's interest to offer a higher equity stake to the VC than the minimum necessary to attract venture capital finance. The key point is that it is in the entrepreneur's interest for the VC to have strong incentives to contribute to the venture. (Correspondingly, the VC has an incentive to be sensitive to the motivational importance of leaving the entrepreneur with a sufficiently high share of the venture's returns.)

A second managerial implication, from the VC's point of view, arises from the observation that the key to good performance is to invest in ventures that allow the VC to earn rents — returns over and above the competitive normal return. This will tend to be the case when VC managerial contributions are highly significant. As a pure financial intermediary, the VC's

equity-based contracts normally will be dominated by banks and other sources of finance and, at best, the VC can anticipate only a normal competitive expected return. Only if it can provide value from its proprietary managerial skill is it likely to earn above-normal returns.

The other and perhaps more surprising implication is that the VC is likely to do better if it can avoid investments where the entrepreneur's input is highly important. In order for VC investments to be successful, the VC needs to have a high enough equity share to have appropriate incentives. However, this equity share will necessarily dilute the entrepreneur's incentive to provide appropriate effort and will also create potential problems or conflicts arising from the entrepreneur's loss of control. If the entrepreneur is crucially important to the venture, then these costs are much more significant than if the entrepreneur is less important to the venture's success.

Another simplification in our analysis is that the financial instrument we consider for the VC is common equity. In practice, VCs use several forms of equity, especially preferred equity.[5] Our analysis has insights to offer regarding the use of these more sophisticated forms. Specifically, one advantage of preferred equity is that it preserves entrepreneurial incentives more effectively than common equity. Because returns to preferred equity are paid before returns to common equity, the entrepreneur is the true residual claimant in the sense that the entrepreneur's return is based on what is left after required returns to preferred equity are paid. The entrepreneur therefore faces relatively strong incentives. We might expect to see preferred equity favored when the effort of the entrepreneur is relatively important.

Finally, we use a very simple form of uncertainty in which there are only two possible states of nature: "success" or "failure," and these states have exogenous probabilities. More generally, we might expect the probability of success to be affected by entrepreneurial and VC effort. In such a framework the central trade-off between entrepreneurial incentives and VC incentives would still exist. However, the specific implications for when venture capital finance would be preferred to bank finance might well be modified. Certainly the modeling exercise would be much more challenging.

Interestingly, despite our simple characterization of uncertainty, the probability of success has non-neutral effects on financing in the sense that the lower success probabilities tend to favor bank finance relative to venture capital finance. In other words, VC "cherry-picking" in our analysis takes the form of VCs favoring projects with a relatively high success probability even more than banks would. The reasoning is that banks always get just the normal or competitive expected return. VCs, on the other hand, due to their ownership of a scarce productive input (their own managerial effort), can earn rents from very good projects and therefore can receive more than the competitive rate of return when success probabilities are high.

This insight suggests another form of cherry-picking that coincides with an important theme in venture capital finance. Specifically, VCs also would be expected to differentially favor ventures with a very strong "best case" potential, even if the probability of success is low. Thus, we might well see VCs financing high variance projects — projects that are long shots but that will provide a very big payoff if they are successful. From a bank's point of view, a big success (sometimes referred to as a "home run") is no better than a modest success — all the

[5] Several authors have examined the common use of convertible preferred securities in venture capital investments in the United States. For an elegant theoretical treatment of this topic, see for example Schmidt (2003).

bank gets is the principal and interest. To a VC, its equity stake allows it to share fully in the success of a "home run" project and earn rents from such projects. In short, ventures like Microsoft, Federal Express, and Intel were more attractive investments to VCs than to banks.

This high variance "cherry-picking" is consistent with the spirit of our analysis but is, strictly speaking, beyond the scope of our simple model. It could, however, be captured by a slight extension. Instead of allowing for just "success" and "failure," we could allow for a richer characterization of uncertainty as, for example, with three states — failure, "modest success," and "major success." We then would generate "high variance" (and "high skewness") cherry-picking effects. Banks would have no particular preference for major success over modest success and therefore would focus simply on low failure probabilities. VCs, on the other hand, would be more willing to trade-off a higher failure probability for some chance of a major success.

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Appendix A. Proofs of propositions

Proof of Proposition 1. The proofs of statements i) and ii) follow from differentiation of (2) and (3), respectively. □

Proof of Proposition 2. i) Follows from a comparison of Eqs. (2) and (9).
ii) Follows from differentiation of Eq. (9).
iii) Follows from differentiation of Eq. (7). □

Proof of Proposition 3. The solution for s is given by Eq. (12). Differentiating with respect to α and β and using Eq. (12) yields:

$$\text{i) } ds/d\alpha = -2\alpha\beta^2/(\alpha^2-2\beta^2)^2<0, \text{ and} \tag{A1}$$

$$\text{ii) } ds/d\beta = 2\alpha^2\beta/(\alpha^2-2\beta^2)^2>0. \tag{A2}$$

iii) From Eq. (12) as α approaches 0 and/or β approaches infinity, s approaches $1/2$. □

Proof of Proposition 4. i) Differentiating Eq. (10) with respect to s yields:

$$\begin{aligned} dV/ds &= p^2\alpha^2 + p^2(\beta^2-2\alpha^2)s \\ &= p^2\alpha^2(1-2s) + p^2\beta^2 \qquad \text{(A3)} \\ &= p^2\alpha^2(1-s) + p^2(\beta^2-\alpha^2)s \qquad \text{(A4)} \end{aligned}$$

Expression (A4) is positive if $s \leq 1/2$. We know from Proposition 3, part iii) that s cannot exceed $1/2$ at an unconstrained solution for s. Therefore dV/ds must be positive at such a solution.

ii) This follows from the observation that Eq. (A4) must be positive if condition C3 is satisfied. □

Proof of Proposition 5. i) Starting with expression (10) and using subscripts to denote partial derivatives:

$$dV/d\beta = V_s ds/d\beta + V_\beta \tag{A5}$$

where $V_s = p^2\alpha^2 + p^2 s(\beta^2 - 2\alpha^2)$.

We can see that $V_s > 0$ for sufficiently large β. In addition, from Eq. (A2) we have $ds/d\beta > 0$ and partial differentiation of Eq. (10) with respect to β yields $V_\beta = p^2 s^2 \beta > 0$. It follows from these facts and Eq. (A5) that $dV/d\beta$ is positive for sufficiently large values of β and that V grows without bound as β gets sufficiently large and must become positive as β becomes sufficiently large.

ii) This follows immediately from differentiating Eq. (10) with respect to θ.

iii) Taking the derivative of Eq. (10) with respect to p (holding other things constant) yields $dV/dp = 2p\alpha^2 s + p(\beta^2 - 2\alpha^2)s^2 = 2p\alpha^2 s(1-s) + p\beta^2 s$, which must be positive. □

Proof of Proposition 6. We assume that conditions C2 and C4 hold. From Eq. (10) the VC's payoff can be written as:

$$V = p^2(\beta^2/2 - \alpha^2)s^2 + p^2\alpha^2 s - \theta \tag{A6}$$

The discriminant can be expressed as $p^2\alpha^4 + 2\theta(\beta^2 - 2\alpha^2)$, which is strictly positive under C4. Therefore there are two real solutions to the binding participation constraint when $\beta^2 \neq 2\alpha^2$.

Remarks. a) $V < 0$ at $s = 0$.

b) The slope of V can be written as $2p^2(\beta^2/2 - \alpha^2)s + p^2\alpha^2$. It is strictly positive at $s = 0$ and strictly positive at $s = 1$ if and only if $\beta^2 > \alpha^2$.

c) V is strictly positive at $s = 1$ if and only if $p^2\beta^2/2 > \theta$, which holds if C2 and C4 hold.

Proposition 6 can now be proven by separating the range of possibilities into four cases.

Case I. $\alpha^2/2 < \beta^2 < \alpha^2$

It is easy to see from Eq. (A1) that V is a strictly concave parabola. From Remarks a), b), and c), we can deduce that either both roots (s_1 and s_2) are between 0 and 1 or only the smaller root s_1 is between 0 and 1. Since $\beta^2 < \alpha^2$, the VC is less productive than the entrepreneur, so in the unconstrained case, the entrepreneur would choose $s = 0$. Since this

would not satisfy the VC's participation constraint, the entrepreneur offers the minimum feasible value of s, i.e., s_1.

Case II. Let $\alpha^2 \leq \beta^2 < 2\alpha^2$

V is still a strictly concave parabola and, from Remarks a), b), and c), we can deduce that only the smaller root s_1 is between 0 and 1. Therefore, V must be strictly increasing in s around s_1. This implies that *an increase in* β, which increases V for a given s, *must lower* s_1, since at the initial s_1 we can see that V is now positive. The limit of V when β^2 tends to $2\alpha^2$ (from below) is $p^2\alpha^2 s - \theta$, which implies that the limit of s_1 when β^2 tends to $2\alpha^2$ is $\theta/(p^2\alpha^2)$.

The unconstrained solution s^* increases with β and its limit when β^2 tends to $2\alpha^2$ is $1/3$. Then $\lim s_1 < \lim s^*$ if and only if $(p^2\alpha^2)/3 > \theta$, which holds under C4.

Therefore, there must exist a $\beta = \beta^*$ with $\alpha^2 \leq \beta^{*2} < 2\alpha^2$, such that the constrained solution s_1 is the optimum when $\alpha^2 \leq \beta^2 < \beta^{*2}$ and the unconstrained solution s^* is the optimum when $\beta^{*2} \leq \beta^2 < 2\alpha^2$.

Case III. $\beta^2 = 2\alpha^2$

V is linear and strictly increasing in s and $s_1 = \theta/(p^2\alpha^2)$. We have already shown that $s^* > \theta/(p^2\alpha^2)$, and hence the unconstrained solution is the optimum.

Case IV. $\beta^2 > 2\alpha^2$

V is a strictly convex parabola with two real roots. From Remarks a), b), and c), it can be easily deduced that the smaller root s_1 is negative and the larger root s_2 is between 0 and 1. Therefore, V is negative for $s < s_2$, and positive for $s \geq s_2$; moreover, it is increasing in s in the neighbourhood of s_2. An increase in β must therefore lower s_2, which is at a minimum when β^2 tends to $2\alpha^2$ from above. The limit of s_2 when β^2 tends to $2\alpha^2$ from above is $\lim s_2 = \theta/(p^2\alpha^2)$. Since s^* increases with β and its limit when β^2 tends to $2\alpha^2$ from above is $\lim s^* = 1/3 > \theta/(p^2\alpha^2)$, the unconstrained solution s^* must be superior to s_2 for all $\beta^2 > 2\alpha^2$.

These four cases show that given conditions C2 and C4, there exists a threshold level of $\beta = \beta^*$ such that:

i) For all values of β in $(\alpha/\sqrt{2}, \beta^*)$, the unique solution to the venture capital finance problem is the constrained optimum $s = (\alpha^2 p - (p^2\alpha^4 - 4\theta\alpha^2 + 2\theta\beta^2)^{1/2})/(p(2\alpha^2 - \beta^2))$.
ii) For all values of $\beta \geq \beta^*$, the unique solution to the venture capital finance problem is the unconstrained optimum $s = (\beta^2 - \alpha^2)/(2\beta^2 - \alpha^2)$. □

Proof of Proposition 7. i) We divide the proof of part (i) into two cases, starting with the case where $\alpha^2/2 < \beta^2$. We consider constrained and unconstrained solutions separately. If the solution for s is unconstrained (in which case condition C4 is the second-order condition), s is given by Eq. (12). Substituting this into expression (15) yields:

$$\Delta E = (p^2/2)(\beta^2 - \alpha^2)^2/(2\beta^2 - \alpha^2) + \theta. \tag{A7}$$

From C2 it follows that $2\beta^2-\alpha^2>0$, which implies immediately that expression (A7) is positive, showing that E must exceed E^B (i.e., that venture capital finance must dominate) in this case.

If the solution for s is constrained by the VC participation constraint, it follows from Eq. (10) that $sp(\alpha e+\beta i)=i^2/2+\theta$. Substituting this in the expression for the entrepreneur's payoff given by Eq. (9) allows us to write the entrepreneur's payoff under venture capital finance as $E=p(\alpha e+\beta i)-(e^2+i^2)/2-\theta$.

Substituting for i and e from Eqs. (7) and (9) and rearranging yields $E=E^B+s(p^2\beta^2-s(p^2\alpha^2+p^2\beta^2)/2$, where E^B is the entrepreneur's payoff from bank finance as given by Eq. (3). It follows that E exceeds E^B (i.e., venture capital is preferred to bank finance) if and only if $s<\lambda$, where $\lambda=2p^2\beta^2/(p^2\alpha^2+p^2\beta^2)$. If $\beta=\alpha/\sqrt{2}$, this reduces to $s<2/3$ (i.e. $\lambda=2/3$). Furthermore, λ is increasing in β. By condition C2, $\beta>\alpha/\sqrt{2}$ and we can see that λ must exceed $2/3$ for any admissible β. However, from Eq. (15) we can see that s must be less than $2/3$. It follows that $s<\lambda$ for all admissible values of β, which proves that ΔE is positive (i.e., that venture capital finance is preferred to bank finance in this case).

We now consider the case where $0\le\beta^2\le\alpha^2/2$.

Over this domain, only the constrained solution for venture capital finance can be feasible as the second-order condition for the unconstrained case is not satisfied and the entrepreneur chooses $s=s_1$. We can show (using a method similar to the proof of Proposition 6) that s_1 is strictly decreasing in β over that domain. λ, on the other hand, is strictly increasing in β. We know from above that $s_1<\lambda$ at $\beta=\alpha/\sqrt{2}$, and it can be shown that at $\beta=0$, $s_1>\lambda$ ($0<s_1<1/2$, while $\lambda=0$). Therefore, there must exist a threshold $\beta=\beta^{**}$ with $0\le\beta^{**2}\le\alpha^2/2$, such that bank finance dominates venture capital finance for $\beta<\beta^{**}$ and venture capital finance dominates bank finance for $\beta\ge\beta^{**}$.

ii) This follows directly from differentiation of Eq. (A7) with respect to p, θ, α, and β. □

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